Probability: Random Variables

Expectation and Variance Problems

In this document we solve two problems involving expectations and variances.

Example 1

The distribution of grades in a statistics course for the past 10 years (A=5,B=4, etc.) are shown in the table:

Grade	1	2	3	4	5
Probability	0.07	0.09	0.34	0.32	0.18

Table 1: Probability Table for the distribution of Grades

Calculate the average and standard deviation of the grade distribution.
The graph of probability mass function for the grades looks like that:

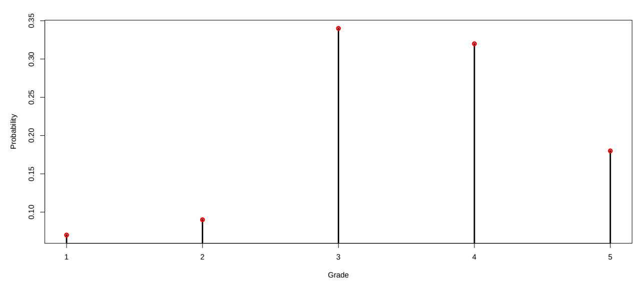

Figure 1: Probability Mass Function of Grades

Using a standard formula for the expectation (we frequently represent expectation as Greek letter μ ('mu')) we get:

$$E(\text{Grade}) = \mu = 1 \times 0.07 + 2 \times 0.09 + 3 \times 0.34 + 4 \times 0.32 + 5 \times 0.18 = 3.45$$

The variance which is sometimes called σ^2 ('sigma' squared) is

$$Var(\text{Grade}) = \sigma^2 = (1 - 3.45)^2 \times 0.07 + (2 - 3.45)^2 \times 0.09 + (3 - 3.45)^2 \times 0.34 +$$
$$+ (4 - 3.45)^2 \times 0.32 + (5 - 3.45)^2 \times 0.18 = 1.21$$

Thus the standard deviation σ is

$$\sigma = \sqrt{\sigma^2} = \sqrt{1.21} = 1.10$$

Example 2

The probability of payoffs in a lottery is shown in the next table:

Payoff	\$0	\$5	\$10	\$20
Probability	0.9	0.07	0.02	0.01

Table 2: Probability Table for the distribution of Payoffs

If the lottery sells 1000 tickets for \$10 each then what is the average amount that the lottery will profit ?

The graph of probability mass function for the payoffs looks like that:

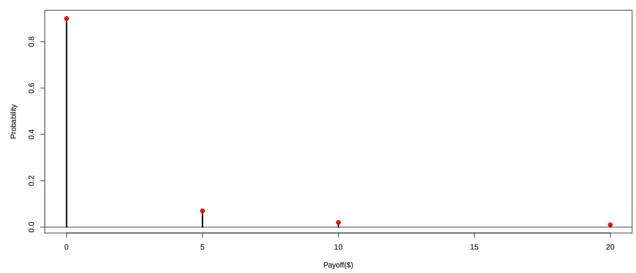

Figure 2: Probability Mass Function of Payoffs

First let's calculate the average payoff using expected value formula:

$$\mu = 0 \times 0.9 + 5 \times 0.07 + 10 \times 0.02 + 20 \times 0.01 = 0.75$$

If the lottery sells 1000 tickets for \$10 each, then the lottery will bring in \$10000.
The average payoff on 1000 tickets will be 1000 tickets $\times$ \$0.75 per ticket = \$750.
Therefore

$$\text{Profit} = \$10000 - \$750 = \$9250$$

To finish this document let's compare two expectations from previous examples. In the 'Grades' example the expectation was 3.45 which is between 3 and 4. That makes sense because the probability of 1,2 and 3 is 50% $(0.07 + 0.09 + 0.34)$ and another 50% is on 4 and 5 so the center should be between around 3 and 4.
In the lottery example the expectation is only \$0.75 which in near 0, since the probability of 0 is 90%. So almost all of the probability is at 0 and therefore the expectation should also be close to 0.