Probability: Random Variables

Variance of Discrete Random Variables

In this document we introduce a concept of variance for discrete random variables.

Consider a fair die example and let

X = face value of a die toss

Then for this random variable we have the following probability distribution:

Value of X	1	2	3	4	5	6
Prob	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Table 1: Probability Table for the fair dice

We already know that expected value $E(X)$ is 3.5.
The variance of X is average deviation from the expected value and for this problem is equal to:

$$Var(X) = (1-3.5)^2\frac{1}{6} + (2-3.5)^2\frac{1}{6} + \cdots + (6-3.5)^2\frac{1}{6} = 2.92$$

Standard deviation is just square root of the variance:

$$SD(X) = \sqrt{Var(X)} = \sqrt{2.92} = 1.71$$

The general formula for the variance of discrete random variable X that can take k values $x_1, x_2, , x_k$ with corresponding probabilities $P(X = x_1), P(X = x_2), , P(X = x_k)$ is

$$Var(X) = (x_1 - E(X))^2\, P(X = x_1) + (x_2 - E(X))^2\, P(X = x_2) + \cdots + (x_k - E(X))^2\, P(X = x_k)$$

$$= \sum_{i=1}^{i=k} (x_i - E(X))^2\, P(X = x_i)$$

In our die example $k = 6$, $x_1 = 1, x_2 = 2, , x_6 = 6$, $E(X) = 3.5$ and $P(X = x_1) = P(X = x_2) = \cdots = P(X = x_6) = \frac{1}{6}$.