Probability: Events

Addition Rules

In this document we show how to calculate probability of an event that is defined in terms of two other events. Let's introduce two events A and B

A = { event that it is raining tomorrow }
B = { event that it is cloudy tomorrow }

What is the probability that it is raining **or** cloudy tomorrow (P(A or B)) ?

In probability **or** means that A can occur or B can occur or both can occur.

Example
Suppose we toss a fair coin twice. What is the probability of a head on the first toss **or** a head on the second toss ?

First we define two corresponding events:

A={Head on the first toss}={HH,HT}
B={Head on the second toss}={HH,TH}

We can picture these events using a 'Venn diagram'. The box represents all the possible outcomes, and circles represent two events A and B:

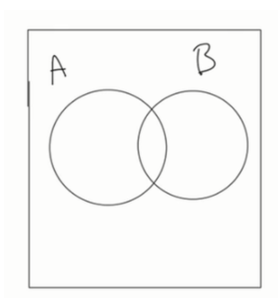

Figure 1: Venn diagram of two events

Since we know outcomes in each event we can draw

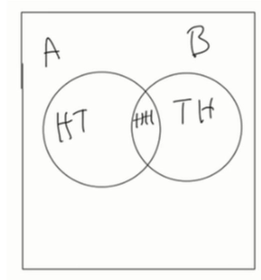

Figure 2: Venn diagram of two events with outcomes

Note that HH is in the overlap of these events since it is in both of them. This region of the overlap is called 'A and B' event. We are interested in probability of A or B, since we have a total of four possibilities {HT,TT,HH,TH} we get:

$P(\text{A or B}) = \frac{3}{4} = \frac{2+2-1}{4} = \frac{2}{4} + \frac{2}{4} - \frac{1}{4} = P(\text{A}) + P(\text{B}) - P(\text{A and B})$

The above result is known as *Addition Rule*.

Example

Probability is often calculated as area under a curve. Suppose the next density represents the distribution of birth weights of Canadian males in grams

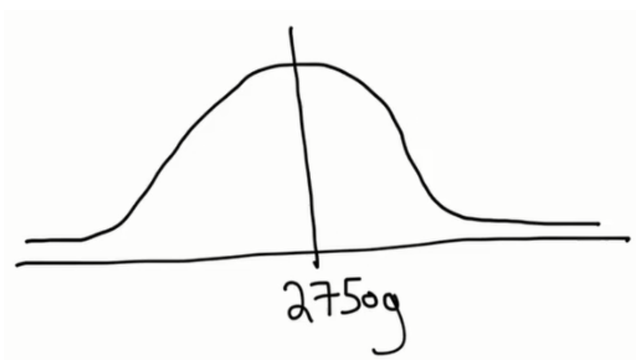

Figure 3: Density curve of Birth weights

What is the probability that a new born male will have a birth weight below 1600g or above 3750g ?

We have the probabilities of being below 1600g and above 3750g:

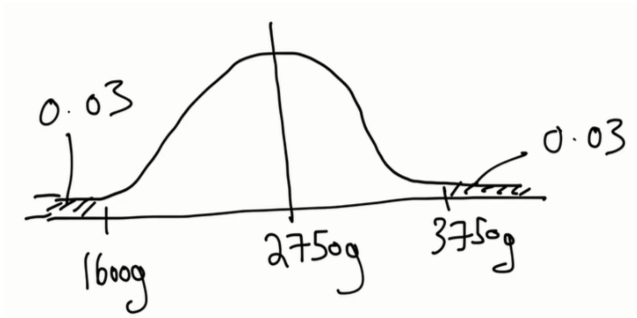

Figure 4: Tail probabilities for the Birth Weight example

Next we introduce two events:
A={Birth Weight $\leq$ 1600g}
B={Birth Weight $\geq$ 3750g}

Now using Addition Rule we get

P(A or B)= P(A)+P(B)−P(A and B)= $0.03 + 0.03 - 0 = 0.06$

Note that P(A and B) is zero because A and B are disjoint (a baby cannot weigh less than 1600g and more than 3750g simultaneously).